

## GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES SOME CLASSES OF 4-TOTAL PRIME CORDIAL LABELING OF GRAPHS

R. Ponraj<sup>\*1</sup>, J.Maruthamani<sup>2</sup> & R.Kala<sup>3</sup>

<sup>\*1</sup>Department of Mathematics, Sri Paramakalyani college, Alwarkurichi-627412, Tamilnadu, India.

<sup>\*2</sup> Research Scholar , Register Number: 18124012091054, Department of mathematics, Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.

<sup>\*3</sup>Department of Mathematics, Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India

### ABSTRACT

Let  $G$  be a  $(p, q)$  graph. Let  $f : V(G) \rightarrow \{1, 2, \dots, k\}$  be a map where  $k \in \mathbb{N}$  and  $k > 1$ . For each edge  $uv$ , assign the label  $\gcd(f(u), f(v))$ .  $f$  is called  $k$ -Total prime cordial labeling of  $G$  if  $|t_f(i) - t_f(j)| \leq 1$ ,  $i, j \in \{1, 2, \dots, k\}$  where  $t_f(x)$  denotes the total number of vertices and the edges labelled with  $x$ . A graph with a  $k$ -total prime cordial labeling is called  $k$ -total prime cordial graph. In this paper we investigate the 4-total prime cordial labeling of some graphs like flower graph, gear graph,  $IT_n \odot K_1$

**Key words:** (Corona, gear graph, Flower graph).

### I. INTRODUCTION

Graphs considered here are finite, simple and undirected. Ponraj et al. [4], have been introduced the concept of  $k$ -total prime cordial labeling and the  $k$ -total prime cordial labeling of certain graphs have been investigated. Also in [4, 5, 6, 7, 8, 9], the 4-total prime cordial labeling behaviour of path, cycle, star, bistar, some complete graphs, comb, double comb, triangular snake, double triangular snake, ladder, friendship graph, jelly fish, book, irregular triangular snake, triangular ladder, armed crown, shadow graph,  $P_n \odot K_2$ ,  $T_n \odot K_2$  and subdivision of some graphs like comb, double comb, star, bistar, triangular snake, ladder, double triangular snake, jelly fish, triangular ladder,  $T_n \odot K_1$  have been investigated. In this paper we investigate the 4-total prime cordial labeling of few graphs like flower graph, gear graph,  $IT_n \odot K_1$ .

### II. PRELIMINARIES

**Definition 2.1.** Let  $G$  be a  $(p, q)$  graph. Let  $f : V(G) \rightarrow \{1, 2, \dots, k\}$  be a function where  $k \in \mathbb{N}$  and  $k > 1$ . For each edge  $uv$ , assign the label  $\gcd(f(u), f(v))$ .  $f$  is called  $k$ -Total prime cordial labeling of  $G$  if  $|t_f(i) - t_f(j)| \leq 1$ ,  $i, j \in \{1, 2, \dots, k\}$  where  $t_f(x)$  denotes the total number of vertices and the edges labelled with  $x$ . A graph with a  $k$ -total prime cordial labeling is called  $k$ -total prime cordial graph.

**Definition 2.2.** Let  $G_1, G_2$  respectively be  $(p_1, q_1), (p_2, q_2)$  graphs. The corona of  $G_1$  with  $G_2$  is the graph  $G_1 \odot G_2$  obtained by taking one copy of  $G_1$ ,  $p_1$  copies of  $G_2$  and joining the  $i$ th vertex of  $G_1$  by an edge to every vertex in the  $i$ th copy of  $G_2$  where  $1 \leq i \leq p_1$ .

**Definition 2.3.** The graph irregular triangular snake  $IT_n$  ( $n \geq 4$ ) is obtained by the path  $P_n : u_1 u_2 \dots u_n$  with vertex set  $V(IT_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n-2\}$  and edge set  $E(IT_n) = E(P_n) \cup \{u_i v_i, u_{i+2} v_i : 1 \leq i \leq n-2\}$ .

**Definition 2.4.** The graph  $W_n = C_n + K_1$  is called a wheel. In a Wheel, a vertex of degree 3 on the cycle is called a rim vertex. A vertex which is adjacent to all the rim vertices is called the central vertex. The edges with one end incident with a rim vertex and the other incident with the central vertex are called spokes.

Definition 2.5. The Helm  $H_n$  is obtained from a wheel  $W_n$  by attaching a pendent edge at each vertex of the cycle  $C_n$ .

Definition 2.6. A Flower graph  $F_n$  is the graph obtained from a Helm by joining each pendent vertex to the central vertex of the Helm.

s

Definition 2.7. The Gear graph  $G_n$  is obtained from the wheel  $W_n$  by adding a vertex between every pair of adjacent vertices of the cycle  $C_n$ .

Theorem 2.8. [4] Every graph is a subgraph of a connected  $k$ -total prime cordial graph.

Theorem 2.9. [4] If  $m \equiv 0 \pmod{k}$ , then  $mG$  is  $k$ -total prime cordial.

Theorem 2.10. [4] If  $G$  is a  $(p, q)$  graph, then  $mG \cup (k-r)K_{1,p+q}$  is  $k$ -total prime cordial, where  $m = kt + r$ ,  $0 \leq r < k$ .

Theorem 2.11. [4] Let  $G_1$  and  $G_2$  be  $(p_1, q_1)$  and  $(p_2, q_2)$ -graph respectively with  $(p_1 + q_1) \equiv 0 \pmod{k}$  and  $(p_2 + q_2) \equiv 0 \pmod{k}$ . If  $G_1$  and  $G_2$  are  $k$ -total prime cordial graph, then the graph  $G_1 \sim G_2$  obtained from  $G_1$  and  $G_2$  by connecting an edge is also a  $k$ -total prime cordial.

Theorem 2.12. [4] Let  $G$  be a  $(p, q)$ - $k$ -total prime cordial graph with  $(p + q) \equiv 0 \pmod{k}$ . Then  $G - e$  is also a  $k$ -total prime cordial graph.

Theorem 2.13. [4] If  $G$  is a  $(p, q)$ - $k$ -total prime cordial graph with  $(p + q) \equiv 0 \pmod{k}$ , then  $G \cup K_2$  is also  $k$ -total prime cordial.

Theorem 2.14. [4] All paths are 4-total prime cordial.

Theorem 2.15. [4] The cycle  $C_n$  is 4-total prime cordial iff  $n \notin \{4, 6, 8\}$ .

Theorem 2.16. [4] If  $n \equiv 0, 7 \pmod{8}$ , then the complete graph  $K_n$  is not 4-total prime cordial.

Theorem 2.17. [4] The star  $K_{1,n}$  is 4-total prime cordial for all  $n$ .

Theorem 2.18. [4] The bistar  $B_{n,n}$  is 4-total prime cordial for all  $n$ .

Theorem 2.19. [4] The join of  $K_2$  with  $mK_1$ ,  $K_2 + mK_1$  is 4-total prime cordial iff  $m \leq 1$ .

Theorem 2.20. [5] The comb  $P_n \odot K_1$  is 4-total prime cordial.

Theorem 2.21. [5] The double comb  $P_n \odot 2K_1$  is 4-total prime cordial.

Theorem 2.22. [5] The graph  $C_n \odot 2K_1$  is 4-total prime cordial.

Theorem 2.23. [5] The ladder  $L_n$  is 4-total prime cordial.

Theorem 2.24. [5] The triangular snake  $T_n$  is 4-total prime cordial.

Theorem 2.25. [5] The double triangular snake  $D(T_n)$  is 4-total prime cordial.

Theorem 2.26. [5] The friendship graph  $C_3(t)$  is 4-total prime cordial iff  $t \equiv 0, 1, 2 \pmod{4}$ .

Theorem 2.27. [6] The subdivision of comb  $S(P_n \odot K_1)$  is 4-total prime cordial.

Theorem 2.28. [6] The subdivision of double comb  $S(P_n \odot 2K_1)$  is 4-total prime cordial.

Theorem 2.29. [6] The subdivision of star  $S(K_1, n)$  is 4-total prime cordial.

Theorem 2.30. [6] The subdivision of bistar  $S(B_n, n)$  is 4-total prime cordial.

Theorem 2.31. [6] The subdivision of triangular snake  $S(T_n)$  is 4-total prime cordial.

Theorem 2.32. [6] The subdivision of ladder  $S(L_n)$  is 4-total prime cordial.

Theorem 2.33. [6] The subdivision of double triangular snake  $S(DT_n)$  is 4-total prime cordial.

Theorem 2.34. [7] The Jelly fish  $J(n, n)$  is 4-total prime cordial for all values of  $n$ .

Corollary 2.34.1. [7] The Jelly fish  $J(m, n)$  is 4-total prime cordial for  $m \neq n$  and  $m, n \geq 8$ .

Theorem 2.35. [7] The irregular triangular snake  $I T_n$  is 4-total prime cordial for  $n \geq 4$ .

Theorem 2.36. [8] The armed crown graph  $AC_n$  is 4-total prime cordial for all  $n \geq 3$ .

Theorem 2.37. [8] The subdivision of jelly fish  $J(n, n)$ ,  $S(J(n, n))$  is 4-total prime cordial for all values of  $n$ .

Theorem 2.38. [9] If  $n \equiv 1 \pmod{4}$ , then  $P_{n^2}$  is 4-total prime cordial.

Theorem 2.39. [9] The shadow graph  $D_2(P_n)$  is 4-total prime cordial iff  $n \notin \{2, 4\}$ .

Theorem 2.40. [9] The corona of  $T_n$  with  $K_2$ ,  $T_n \odot K_2$  is 4-total prime cordial for all  $n \geq 2$ .

Theorem 2.41. [9] The subdivision of  $T_n$  with  $K_2$  ( $T_n \odot K_1$ ),  $S(T_n \odot K_1)$  is 4-total prime cordial for all  $n \geq 2$ .

Remark. 2- total prime cordial graph is 2-total product cordial graph.

### III. MAIN RESULTS

Theorem 3.1. The flower graph  $F_n$  is 4-total prime cordial iff  $n \neq 3$ .

Proof. Let  $C_n$  be the cycle  $u_1 u_2 \dots u_n u_1$ . Let  $V(F_n) = \{u_i, v_i, w_i : 1 \leq i \leq n\}$  and  $E(F_n) = E(C_n) \cup \{u_i v_i, u_i w_i : 1 \leq i \leq n\}$ . Clearly  $|V(F_n)| + |E(F_n)| = 6n + 1$ .

Case 1.  $n \equiv 0 \pmod{4}$ .

Let  $n = 4r$ ,  $r > 1$  and  $r \in \mathbb{N}$ .

Subcase 1.  $r$  is even.

Assign the label 4 to the central vertex  $u$ . Next assign the label 4 to the vertex  $u_1, u_2, \dots, u_r$  and assign 2 to the vertices  $u_{r+1}, u_{r+2}, \dots, u_{2r}$  then we assign the label 3 to the vertices  $u_{2r+1}, u_{2r+2}, \dots, u_{3r/2+1}$ . Finally we assign the label 1 to the vertices  $u_{3r/2+2}, \dots, u_{4r}$ . Now we consider the vertices  $v_i$  ( $1 \leq i \leq n$ ). Assign the label 4 to the vertices  $v_1, v_2, \dots, v_r$  and assign 2 to the vertices  $v_{r+1}, v_{r+2}, \dots, v_{2r}$  then we assign the label 3 to the vertices  $v_{2r+1}, v_{2r+2}, \dots, v_{3r/2}$ . Finally we assign the label 1 to the vertices  $v_{3r/2+1}, \dots, v_{4r}$ . Here  $tf(1) = tf(2) = tf(4) = 6r$  and  $tf(3) = 6r + 1$ .

Subcase 2.  $r$  is odd.

Assign the label 4 to the central vertex  $u$ . Next assign the label 4 to the vertex  $u_1, u_2, \dots, u_r$  and assign 2 to the vertices  $u_{r+1}, u_{r+2}, \dots, u_{2r}$  then we assign the label 3 to the vertices  $u_{2r+1}, u_{2r+2}, \dots, u_{7r-1/2+1}$ . Finally we assign the label 1 to the vertices  $u_{7r-1/2+2}, \dots, u_{4r}$ . Now we consider the vertices  $v_i (1 \leq i \leq n)$ . Assign the label 4 to the vertices  $v_1, v_2, \dots, v_r$  and assign 2 to the vertices  $v_{r+1}, v_{r+2}, \dots, v_{2r}$  then we assign the label 3 to the vertices  $v_{2r+1}, v_{2r+2}, \dots, v_{7r-1/2+1}$ . Finally we assign the label 1 to the vertices  $v_{7r-1/2+2}, \dots, v_{4r}$ . Clearly  $tf(1) = tf(2) = tf(4) = 6r$  and  $tf(3) = 6r + 1$ .

Case 2.  $n \equiv 1 \pmod{4}$ .

Let  $n = 4r + 1, r > 1$  and  $r \in \mathbb{N}$ .

Subcase 1.  $r$  is even.

As in subcase(1) of case 1, assign the label to the vertices  $u, u_i (1 \leq i \leq n - 1)$  and  $v_i (1 \leq i \leq n - 2)$ . Finally we assign the labels 1, 2, 4 to the vertices  $u_{4r}, v_{4r-1}$  and  $v_{4r}$  respectively. Clearly  $tf(1) = tf(2) = tf(4) = 6r + 2$  and  $tf(3) = 6r + 1$ .

Subcase 2.  $r$  is odd.

As in subcase(2) of case 1, assign the label to the vertices  $u, u_i (1 \leq i \leq n - 1)$  and  $v_i (1 \leq i \leq n - 2)$ . Finally we assign the labels 1, 2, 4 to the vertices  $u_{4r}, v_{4r-1}$  and  $v_{4r}$  respectively. Obviously  $tf(1) = tf(2) = tf(4) = 6r + 2$  and  $tf(3) = 6r + 1$ .

Case 3.  $n \equiv 2 \pmod{4}$ .

Let  $n = 4r + 2, r > 1$  and  $r \in \mathbb{N}$ .

Subcase 1.  $r$  is even.

As in subcase(1) of case 2, assign the label to the vertices  $u, u_i (1 \leq i \leq n - 2)$  and  $v_i (1 \leq i \leq n - 2)$ . Finally we assign the labels 3, 4, 2, 3 respectively to the vertices  $u_{4r-1}, u_{4r}, v_{4r-1}$  and  $v_{4r}$ . Clearly  $tf(1) = tf(3) = tf(4) = 6r + 3$  and  $tf(2) = 6r + 4$ .

Subcase 2.  $r$  is odd.

As in subcase(2) of case 1, assign the label to the vertices  $u, u_i (1 \leq i \leq n - 3)$  and  $v_i (1 \leq i \leq n - 3)$ . Finally we assign the labels 3, 2, 1, 4, 4, to the vertices  $u_{4r-2}, u_{4r-1}, u_{4r}, v_{4r-2}, v_{4r-1}$  and  $v_{4r}$  respectively. It is easy to verify that  $tf(1) = tf(2) = tf(3) = 6r + 3$  and  $tf(4) = 6r + 4$ .

Case 4.  $n \equiv 3 \pmod{4}$ .

Let  $n = 4r + 3, r > 1$  and  $r \in \mathbb{N}$ .

Subcase 1.  $r$  is even.

As in subcase(1) of case 3, assign the label to the vertices  $u, u_i (1 \leq i \leq n - 4)$  and  $v_i (1 \leq i \leq n - 4)$ . Finally we assign the labels 2, 3, 4, 3, 2, 3, 1, 4 respectively to the vertices  $u_{4r-3}, u_{4r-2}, u_{4r-1}, u_{4r}, v_{4r-3}, v_{4r-2}, v_{4r-1}$  and  $v_{4r}$ . Here  $tf(1) = 6r + 4$  and  $tf(2) = tf(3) = tf(4) = 6r + 5$ .

Subcase 2.  $r$  is odd.

As in subcase(2) of case 3, assign the label to the vertices  $u, u_i (1 \leq i \leq n - 2)$  and  $v_i (1 \leq i \leq n - 2)$ . Finally we assign the labels 1, 3, 3, 2 to the vertices  $u_{4r-1}, u_{4r}, v_{4r-1}$  and  $v_{4r}$  respectively. It is easy to verify that  $tf(1) = tf(2) = tf(3) = 6r + 5$  and  $tf(4) = 6r + 4$ .

Case 5.  $n = 3$ .

Suppose  $f$  is a 4-total prime cordial labeling of  $Fl_3$ , then any one of the following types occur:

Type 1:  $tf(1) = tf(2) = tf(3) = 5$  and  $tf(4) = 4$ .

Type 2:  $tf(2) = tf(3) = tf(4) = 5$  and  $tf(1) = 4$ .

Type 3:  $tf(1) = tf(3) = tf(4) = 5$  and  $tf(2) = 4$ .

Type 4:  $tf(1) = tf(2) = tf(4) = 5$  and  $tf(3) = 4$ .

Subcase 1.

For this, it is easy to verify that 4 must be labeled to 3 consecutive vertices of  $F L_3$ . That is, 4 must be labeled to all the three vertices of an induced subpath  $P_3 v_1 u_2$  of  $F L_3$ . Similarly for  $t_f(3) = 5$ , 3 must be labeled to all the three vertices of another induced subpath  $P_3' v_3 u_3 u_1$  of  $F L_3$  which is disjoint from  $P_3$ . Now we have only one vertex remaining in  $F L_3$ . If this vertex is labeled by 2, then  $t_f(2) > 4$ , a contradiction to Type 3.

Subcase 2.

For Type 2, the proof is symmetry to subcase 1.

Subcase 3.

For this, it is easy to verify that 4 must be labeled to 3 consecutive vertices of  $F L_3$ . That is, 4 must be labeled to all the three vertices of an induced subpath  $P_3 v_1 u_2$  of  $F L_3$ . Similarly for  $t_f(3) = 4$ , 3 must be labeled to all the two vertices of another induced subpath  $P_2 u_3 u_1$  of  $F L_3$  which is disjoint from  $P_3$ . Now we have only two vertices remaining in  $F L_3$ . If any one of the vertex are labeled by 3 which is non-adjacent to the vertex labeled by 3. Now we have only one vertex remaining in  $F L_3$ . If it is labeled by 2, then  $t_f(2) < 5$ , a contradiction to Type 4.

Subcase 4.

For Type 1, the proof is symmetry to subcase 3.

Case 6.  $n = 4, 5, 6, 7$ .

Table 1. A 4-total prime cordial labeling

n	4	5	6	7
u	4	4	4	4
u1	4	4	4	4
u2	2	2	4	4
u3	3	3	2	2
u4	3	3	3	3
u5		3	3	3
u6			3	3
u7				3
v1	4	4	4	4
v2	2	2	2	2
v3	3	3	2	2
v4	3	2	3	3
v5		4	3	4
v6			1	3
v7				2

Theorem 3.2. The gear graph  $G_n$  is 4-total prime cordial iff  $n \neq 3$ .

Proof. Let  $C_n$  be the cycle  $u_1 u_2 \dots u_n u_1$ . Let  $u$  be the central vertex of the cycle  $C_n$ . Let  $v_i$  be the vertex subdivide the edge in the cycle  $C_n$ . Clearly  $|V(G_n)| + |E(G_n)| = 5n + 1$ .

Case 1.  $n \equiv 0 \pmod{4}$ .

Let  $n = 4r$ ,  $r > 1$  and  $r \in \mathbb{N}$ .

Subcase 1.  $r$  is even.

Assign the label 4 to the central vertex  $u$ . Next assign the label 4 to the vertex  $u_1, u_2, \dots, u_r$  and assign 2 to the vertices  $u_{r+1}, u_{r+2}, \dots, u_{2r}$  then we assign the label 3 to the vertices  $u_{2r+1}, u_{2r+2}, \dots, u_{7r/2}$ . Finally we assign the label 1 to the vertices  $u_{7r/2+1}, \dots, u_{4r}$ . Now we consider the vertices  $v_i$  ( $1 \leq i \leq n$ ). Assign the label 4 to the vertices  $v_1, v_2, \dots, v_r$  and assign 2 to the vertices  $v_{r+1}, v_{r+2}, \dots, v_{2r}$  then we assign the label 3 to the vertices  $v_{2r+1}, v_{2r+2}, \dots, v_{3r+1}$ . Finally we assign the label 1 to the vertices  $v_{3r+2}, v_{3r+3}, \dots, v_{4r}$ . Here  $tf(1) = tf(2) = tf(4) = 5r$  and  $tf(3) = 5r + 1$ .

Subcase 2.  $r$  is odd.

Assign the label 4 to the central vertex  $u$ . Next assign the label 4 to the vertex  $u_1, u_2, \dots, u_r$  and assign 2 to the vertices  $u_{r+1}, u_{r+2}, \dots, u_{2r}$  then we assign the label 3 to the vertices  $u_{2r+1}, u_{2r+2}, \dots, u_{7r-1/2}$ . Finally we assign the label 1 to the vertices  $u_{7r-1/2+1}, \dots, u_{4r}$ . Now we consider the vertices  $v_i$  ( $1 \leq i \leq n$ ). Assign the label 4 to the vertices  $v_1, v_2, \dots, v_r$  and assign 2 to the vertices  $v_{r+1}, v_{r+2}, \dots, v_{2r}$  then we assign the label 3 to the vertices  $v_{2r+1}, v_{2r+2}, \dots, v_{3r+1}$ . Finally we assign the label 1 to the vertices  $v_{3r+2}, v_{3r+3}, \dots, v_{4r}$ . Clearly  $tf(1) = 5r + 1$  and  $tf(2) = tf(3) = tf(4) = 5r$ .

Case 2.  $n \equiv 1 \pmod{4}$ .

Let  $n = 4r + 1, r > 1$  and  $r \in \mathbb{N}$ .

Subcase 1.  $r$  is even.

As in subcase(1) of case 1, assign the label to the vertices  $u, u_i$  ( $1 \leq i \leq n - 1$ ) and  $v_i$  ( $1 \leq i \leq n - 2$ ). Finally we assign the labels 3, 2, 4 to the vertices  $u_{4r}, v_{4r-1}$  and  $v_{4r}$  respectively. Clearly  $tf(1) = tf(2) = 5r + 1$  and  $tf(3) = tf(4) = 5r + 2$ .

Subcase 2.  $r$  is odd.

As in subcase(2) of case 1, assign the label to the vertices  $u, u_i$  ( $1 \leq i \leq n - 1$ ) and  $v_i$  ( $1 \leq i \leq n - 2$ ). Finally we assign the labels 3, 2, 4 to the vertices  $u_{4r}, v_{4r-1}$  and  $v_{4r}$  respectively. Obviously  $tf(1) = tf(4) = 5r + 2$  and  $tf(2) = tf(3) = 5r + 1$ .

Case 3.  $n \equiv 2 \pmod{4}$ .

Let  $n = 4r + 2, r > 1$  and  $r \in \mathbb{N}$ .

Subcase 1.  $r$  is even.

Assign the label to the vertices  $u, u_i$  ( $1 \leq i \leq n - 2$ ) and  $v_i$  ( $1 \leq i \leq n - 3$ ) by subcase(1) of case 1. Finally we assign the labels 1, 2, 3, 3, 4 respectively to the vertices  $u_{4r-1}, u_{4r}, v_{4r-2}, v_{4r-1}$  and  $v_{4r}$ . Clearly  $tf(1) = tf(2) = tf(3) = 5r + 3$  and  $tf(4) = 5r + 2$ .

Subcase 2.  $r$  is odd.

Assign the label to the vertices  $u, u_i$  ( $1 \leq i \leq n - 2$ ) and  $v_i$  ( $1 \leq i \leq n - 3$ ) by subcase(2) of case 1. Finally we assign the labels 2, 3, 4, 3, 4 to the vertices  $u_{4r-1}, u_{4r}, v_{4r-2}, v_{4r-1}$  and  $v_{4r}$  respectively. It is easy to verify that  $tf(1) = 5r + 2$  and  $tf(2) = tf(3) = tf(4) = 5r + 3$ .

Case 4.  $n \equiv 3 \pmod{4}$ .

Let  $n = 4r + 3, r > 1$  and  $r \in \mathbb{N}$ .

Subcase 1.  $r$  is even.

As in subcase(1) of case 2, assign the label to the vertices  $u, u_i$  ( $1 \leq i \leq n - 3$ ) and  $v_i$  ( $1 \leq i \leq n - 4$ ). Finally we assign the labels 4, 3, 2, 4, 3, 2, 1 respectively to the vertices  $u_{4r-2}, u_{4r-1}, u_{4r}, v_{4r-3}, v_{4r-2}, v_{4r-1}$  and  $v_{4r}$ . Clearly  $tf(1) = tf(2) = tf(3) = tf(4) = 5r + 4$ .

Subcase 2.  $r$  is odd.

As in subcase(2) of case 2, assign the label to the vertices  $u, u_i$  ( $1 \leq i \leq n - 3$ ) and  $v_i$  ( $1 \leq i \leq n - 3$ ). Finally we assign the labels 2, 4, 3, 3, 4, 3 to the vertices  $u_{4r-2}, u_{4r-1}, u_{4r}, v_{4r-2}, v_{4r-1}$  and  $v_{4r}$  respectively. It is easy to verify that  $tf(1) = tf(2) = tf(3) = tf(4) = 5r + 4$ .

Case 5.  $n = 3$ .

Suppose  $f$  is a 4-total prime cordial labeling of  $G_3$ , then  $tf(1) = tf(2) = tf(3) = tf(4) = 4$ .

Subcase 1.

4 is the labels of the central vertex  $u$ . Suppose 4 is label of the adjacent rim vertices, then  $tf(4) > 4$ , a contradiction.

Subcase 1(a).

4 is the labels of the central vertex  $u$ . Suppose 4 is label of the non-adjacent rim vertices, then  $tf(4) = 4$ ,  $tf(3) < 4$  and  $tf(2) > 4$ , a contradiction.

Subcase 2.

In this case, we get the label  $tf(4) = 4$ . Suppose 4 is label of the consecutive rim vertices, then  $tf(4) > 4$ , a contradiction.

Subcase 2(a).

Suppose 4 is label to the not consecutive rim vertices, then  $tf(4) < 4$ ,  $tf(3) < 4$  and  $tf(1) > 4$ , a contradiction.

Subcase 2(b).

Suppose 4 is label to the non-adjacent rim vertices, then  $tf(2) < 4$  and  $tf(1) > 4$ , a contradiction.

Case 6.  $n = 4, 5, 6, 7$ .

Table 2. A 4-total prime cordial labeling

n	4	5	6	7
u	4	4	4	4
u1	4	4	4	4
u2	2	2	4	4
u3	3	4	2	2
u4	3	3	3	3
u5		3	3	3
u6			1	3
u7				2
v1	4	4	4	4
v2	2	2	2	2
v3	3	3	2	2
v4	1	3	3	4
v5		1	3	3
v6			3	3
v7				1

Theorem 3.3. The corona of  $IT_n$  with  $K_1$ ,  $IT_n \odot K_1$  is 4-total prime cordial for all  $n \geq 4$ .

Proof. Let  $P_n$  be the path  $u_1, u_2, \dots, u_n$ . Let  $v_i$  be the vertex which is adjacent to both  $u_i$  and  $u_{i+2}$  ( $1 \leq i \leq n-2$ ). Let  $x_i$  be the pendant vertices adjacent to  $u_i$  ( $1 \leq i \leq n$ ). Let  $y_i$  be the pendant vertices adjacent to  $v_i$  ( $1 \leq i \leq n-2$ ). It is easy to verify that  $|V(I \cap K_1)| + |E(I \cap K_1)| = 9n - 11$ .

Case 1.  $n \equiv 0 \pmod{4}$ .

Let  $n = 4r$ ,  $r > 1$  and  $r \in \mathbb{N}$ . Assign the label 4 to the vertices  $u_1, u_2, \dots, u_r$  and assign the label 2 to the vertices  $u_{r+1}, u_{r+2}, \dots, u_{2r}$ . Next we assign the label 3 to the vertices  $u_{2r+1}, u_{2r+2}, \dots, u_{3r}$  then we assign 1 to the vertices  $u_{3r+1}, u_{3r+2}, \dots, u_{4r-2}$ . Finally we assign the labels 4, 3 to the vertices  $u_{4r-1}$  and  $u_{4r}$  respectively. Next we consider the vertices  $v_i$  ( $1 \leq i \leq n-2$ ). Assign the label 4 to the vertices  $v_1, v_2, \dots, v_r$  and assign the label 2 to the vertices  $v_{r+1}, v_{r+2}, \dots, v_{2r-1}$  and we assign the label 3 to the vertices  $v_{2r}, v_{2r+1}, \dots, v_{3r-1}$ . Finally we assign the label 1 to the vertices  $v_{3r}, v_{3r+1}, \dots, v_{4r-2}$ . Now we move to the vertices  $x_i$  ( $1 \leq i \leq n$ ). Assign the label 4 to the vertices  $x_1, x_2, \dots, x_r$  and assign the label 2 to the vertices  $x_{r+1}, x_{r+2}, \dots, x_{2r}$ . Next we assign the label 3 to the vertices  $x_{2r+1}, x_{2r+2}, \dots, x_{3r}$  then we assign 1 to the vertices  $x_{3r+1}, x_{3r+2}, \dots, x_{4r-2}$ . Finally we assign the labels 3, 2 to the vertices  $x_{4r-1}$  and  $x_{4r}$  respectively. Next we consider the vertices  $y_i$  ( $1 \leq i \leq n-2$ ). Assign the label 4 to the vertices  $y_1, y_2, \dots, y_r$  and assign the label 2 to the vertices  $y_{r+1}, y_{r+2}, \dots, y_{2r-1}$  and we assign the label 3 to the vertices  $y_{2r}, y_{2r+1}, \dots, y_{3r-2}$ . Finally we assign the label 1 to the vertices  $y_{3r-1}, y_{3r}, \dots, y_{4r-2}$ . Clearly  $tf(1) = tf(2) = tf(3) = 9r - 3$  and  $tf(4) = 9r - 2$ .

Case 2.  $n \equiv 1 \pmod{4}$ .

Let  $n = 4r + 1$ ,  $r > 1$  and  $r \in \mathbb{N}$ . As in case 1, assign the label to the vertices  $u_i$  ( $1 \leq i \leq n-1$ ),  $v_i$  ( $1 \leq i \leq n-3$ ),  $x_i$  ( $1 \leq i \leq n-1$ ) and  $y_i$  ( $1 \leq i \leq n-3$ ). Finally we assign the labels 3, 2, 4, 4 respectively to the vertices  $u_{4r}, v_{4r-2}, x_{4r}$  and  $y_{4r-2}$  respectively. Here  $tf(1) = tf(3) = 9r - 1$  and  $tf(2) = tf(4) = 9r$ .

Case 3.  $n \equiv 2 \pmod{4}$ .

Let  $n = 4r + 2$ ,  $r > 1$  and  $r \in \mathbb{N}$ . As in case 2, assign the label to the vertices  $u_i$  ( $1 \leq i \leq n-1$ ),  $v_i$  ( $1 \leq i \leq n-3$ ),  $x_i$  ( $1 \leq i \leq n-1$ ) and  $y_i$  ( $1 \leq i \leq n-3$ ). Finally we assign the labels 3, 2, 4, 4 respectively to the vertices  $u_{4r}, v_{4r-2}, x_{4r}$  and  $y_{4r-2}$  respectively. It is easy to verify that  $tf(1) = tf(2) = tf(4) = 9r + 2$  and  $tf(3) = 9r + 1$ .

Case 4.  $n \equiv 3 \pmod{4}$ .

Let  $n = 4r + 3$ ,  $r > 1$  and  $r \in \mathbb{N}$ . As in case 3, assign the label to the vertices  $u_i$  ( $1 \leq i \leq n-1$ ),  $v_i$  ( $1 \leq i \leq n-3$ ),  $x_i$  ( $1 \leq i \leq n-2$ ) and  $y_i$  ( $1 \leq i \leq n-3$ ). Finally we assign the labels 4, 4, 3, 2, 3 respectively to the vertices  $u_{4r}, v_{4r-2}, x_{4r-1}, x_{4r}$  and  $y_{4r-2}$  respectively. Here  $tf(1) = tf(2) = tf(3) = tf(4) = 9r + 4$ .

Case 5.  $n = 4, 5, 6, 7$ .

Table 3. A 4-total prime cordial labeling

n	4	5	6	7
$u_1$	4	4	4	4
$u_2$	2	4	4	4
$u_3$	3	2	2	2
$u_4$	3	3	3	2
$u_5$		3	3	3
$u_6$			3	3
$u_7$				1
$v_1$	4	4	4	4
$v_2$	2	2	2	4
$v_3$		1	1	3
$v_4$			1	3



v5				1
x1	4	4	4	4
x2	2	2	4	4
x3	3	2	2	2
x4	1	3	3	2
x5		3	3	3
x6			3	3
x7				1
y1	4	4	4	4
y2	3	3	2	2
y3		1	2	2
y4			1	3
y5				1

### REFERENCES

1. I.Cahit, *Cordial graphs:A weaker version of graceful and harmonious graphs*, *Ars Combinatoria*, 23(1987), 201-207.
2. J.A.Gallian, *A Dynamic survey of graph labeling*, *The Electronic Journal of Combinatorics*, 19 (2017) #Ds6.
3. F.Harary, *Graph theory*, Addison wesley, New Delhi (1969).
4. R.Ponraj, J.Maruthamani and R.Kala, *k-Total prime cordial labeling of graphs*. (communicated)
5. R.Ponraj, J.Maruthamani and R.Kala, *Some 4-total prime cordial labeling of graphs*. (communicated)
6. R.Ponraj, J.Maruthamani and R.Kala, *4-Total prime cordiality of certain sub-divided graphs*. (communicated)
7. R.Ponraj, J.Maruthamani and R.Kala, *4-total prime cordial labeling of some special graphs*. (communicated)
8. R.Ponraj, J.Maruthamani and R.Kala, *New families of 4-total prime cordial graph*. (communicated)
9. R.Ponraj, J.Maruthamani and R.Kala, *Some results on 4-total prime cordial graph*. (communicated)